Designing Menus of Contracts Efficiently: The Power of Randomization

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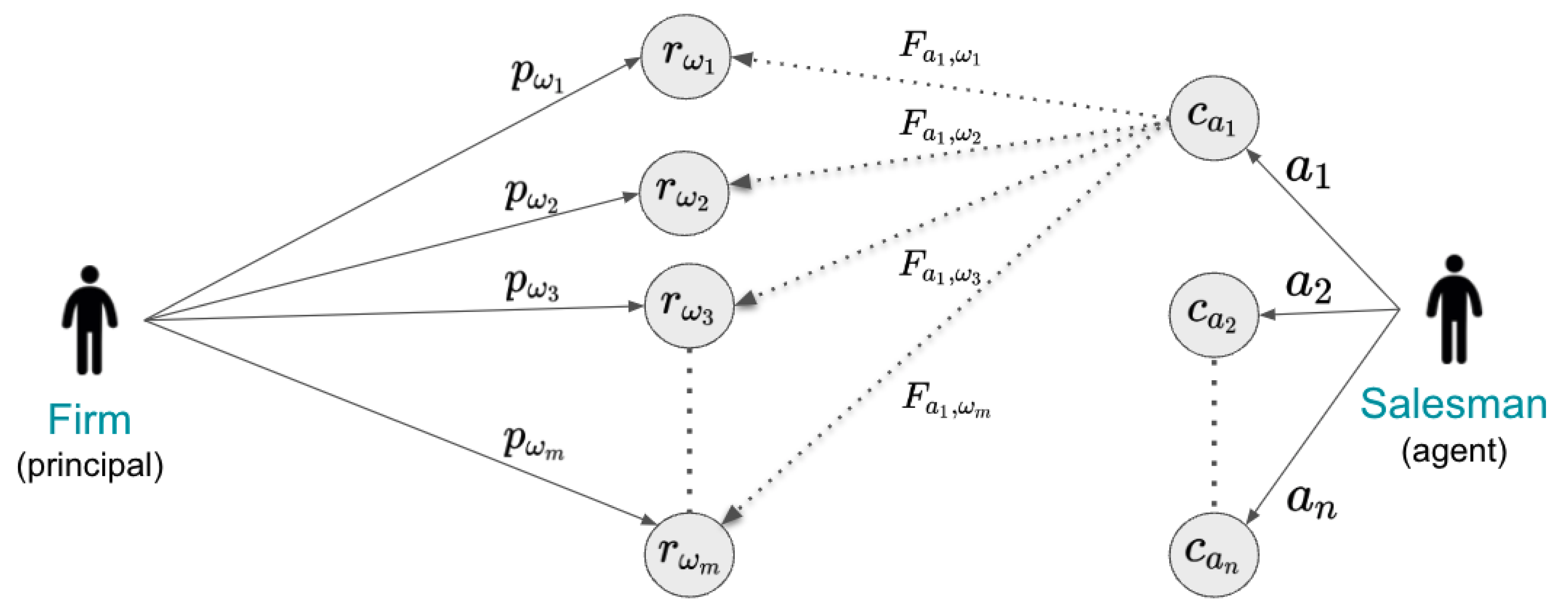
PRINCIPAL-AGENT PROBLEM

- Two players: principal and agent
- The agent chooses among personally-costly actions
- Each action defines a probability distribution over the possible outcomes
- The principal receives a reward that only depends on the outcome
- The action of the agent is hidden and the principal only observes the outcome

The principal's goal is to design an outcome-dependent payment scheme (a.k.a. contract) so as to incentivize the agent to play an action that is profitable for them



PRINCIPAL-AGENT PROBLEM WITH TYPES

- The agent has private information (for example, salesman's experience)
- This is collectively encoded by the agent's type θ, belonging to a finite set Θ
- The type θ is drawn beforehand according to a distribution that is *known* to the principal
- The type θ determines both the probability distributions over outcomes of agent's actions and their costs

Connection with Mechanism Design

The principal-agent interaction goes as follows:

(1) The principal commits to a menu of contracts, one for each possible agent's type
(2) The agent reports (possibly untruthfully) a type
(3) The principal puts in place the contract in the menu corresponding to the reported type

Principal's Computational Problem

Find a utility-maximizing menu of contracts by searching among those that are incentive compatible, i.e., they incentivize the agent to truthfully report their true type to the principal

MAIN RESULT → MENUS OF RANDOMIZED CONTRACTS

- It is known that by using menus of deterministic contracts the principal's problem is **APX**-hard
- Even finding an optimal single contract is largely computationally intractable

We extend the principal's commitment capabilities to menus of randomized contracts, which select payment vectors to be adopted stochastically

Theorem
There exists a polynomial-time algorithm that finds an optimal incentive-compatible menu of randomized contracts for the principal.

- Structural characterization of the space of deterministic contracts over which randomization is needed
- This results in a bound on the maximum payment value needed → Randomization only over a finite but exponentially-sized set of deterministic contracts
- Ellipsoid algorithm to solve a resulting linear programming formulation of the problem with *exponentially-many* variables and polynomially-many constraints

OTHER HARDNESS RESULTS → MENUS OF DETERMINISTIC CONTRACTS

- We tighten the hardness results for the problem of computing an optimal menu of deterministic contracts
- Our result holds even with a constant number of actions and only four outcomes, and it is tight

Theorem
The problem of finding an optimal incentive-compatible menu of deterministic contracts cannot be approximated within any multiplicative factor and it does not admit an additive FPTAS unless **P** = **NP**.

Theorem
The problem of finding an optimal incentive-compatible menu of deterministic contracts admits an additive PTAS when the number of outcomes is constant.

- The PTAS works by first finding an approximately-incentive-compatible menu of deterministic contracts and then transforming it into an incentive-compatible menu by only incurring in a small loss in terms of principal's expected utility
- **Side Results** → with a constant number of types or with only two outcomes, the problem can be solved in polynomial time

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