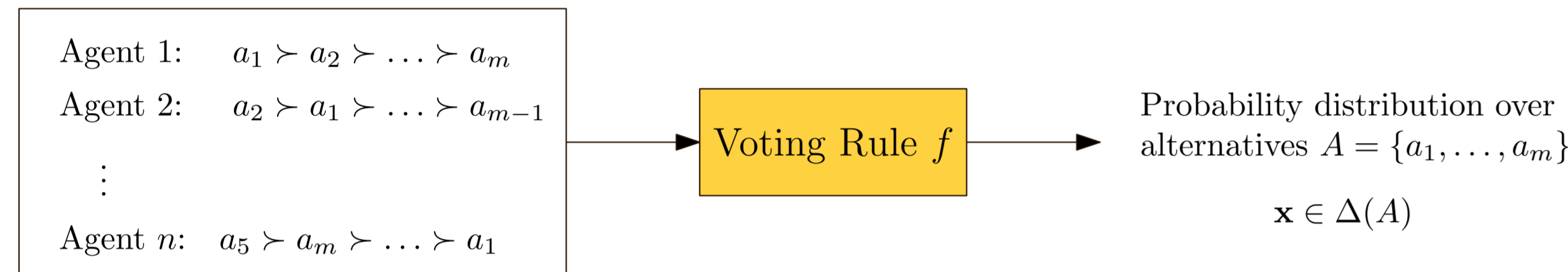


## Single-Winner Randomized Voting

We consider the problem of designing **randomized voting rules** that aggregate **agents' ranked preferences** and arrive at a collective decision with **high social welfare** and which is **fair** to all agents.

- **Input:** Complete rankings over  $m$  alternatives  $A$
- **Output:** Probability distribution over  $A$



**Implicit Utilitarian Voting:** [Procaccia and Rosenschein, 2006]

Assume that each agent  $i$  has a **cardinal utility function**  $u_i : A \rightarrow \mathbb{R}_{\geq 0}$  over alternatives, but **reports only the induced ranking** over alternatives to the voting rule.

**Approach:** We evaluate how well voting rules do, in **worst-case** scenario, on measures of social welfare (**distortion**) and of **proportional fairness**, computed based on the hidden utility functions.

## Distortion

The **Utilitarian Welfare** of distribution  $\mathbf{x} \in \Delta(A)$  is defined as the **sum of agent utilities**:

$$UW(\mathbf{x}, \vec{u}) = \sum_{i \in N} u_i(\mathbf{x}), \quad \text{where } u_i(\mathbf{x}) = \mathbb{E}_{a \sim \mathbf{x}}[u_i(a)].$$

**Definition 1 (Distortion).** The distortion of a voting rule  $f$  over the utility class  $\mathcal{U}$  is

$$\text{Dist}_m(f, \mathcal{U}) = \frac{\max_{\text{utility profiles } \vec{u} \in \mathcal{U}^n} \max_{\text{preference profile } \vec{\sigma} \text{ induced by } \vec{u}} UW(\mathbf{y}, \vec{u})}{UW(f(\vec{\sigma}), \vec{u})},$$

i.e., the worst-case **ratio between the welfare of the optimal outcome and the welfare of  $f(\sigma)$** .

We analyze distortion over different **utility classes**:

- **Unit-sum:**  $\sum_{a \in A} u(a) = 1$ .
- **Unit-range:**  $\max_{a \in A} u(a) = 1$ .
- **Approval:**  $u_i(a) \in \{0, 1\}$  for all  $a \in A$ , with at least one approval. (Subclass of unit-range.)
- **Balanced:**  $\max_{a \in A} u(a) \leq 1$  and  $\sum_{a \in A} u(a) \geq 1$ . (Includes unit-sum, unit-range, approval.)

## Optimal Distortion

**Theorem 1.** For all classes of **unit-sum, unit-range, approval, and balanced** utilities, the **Stable Lottery rule** achieves the **optimal  $\Theta(\sqrt{m})$  distortion**.

Prior to this work, [Boutilier et al., 2015] proposed a rule that achieves  $O(\sqrt{m} \log^*(m))$  distortion for **unit-sum** utilities.

## Stability in Multi-Winner Voting

In multi-winner voting, the problem of selecting a committee  $X \subseteq A$  of  $k$  alternatives, a general notion of fairness is **stability**.

**Idea:** A group of  $\frac{n}{k}$  agents should be able to decide over one of the  $k$  slots in the committee.

For a committee  $X$  and an alternative  $a$ , define  $V(X, a) = |\{i : i \text{ prefers } a \text{ to all of } X\}|$ .

**Stable Committee:**  $X$  is stable if for all  $a \in A$ ,  $V(a, X) < \frac{n}{k}$ .

**Stable Lottery:** A distribution  $\mathbf{X}$  over committees of size  $k$  is stable if for all alternatives  $a \in A$ ,

$$\mathbb{E}_{X \sim \mathbf{X}}[V(a, X)] < \frac{n}{k}.$$

For every preference profile and any  $k$ , **a stable lottery always exists** [Cheng et al., 2020].

## Stable Lottery Rule

**Stable Lottery Rule.** Let  $\mathbf{X}$  be a **stable lottery** over committees of size  $k = \sqrt{m}$ . Then,

- With probability 1/2:
  1. Sample a committee  $X \sim \mathbf{X}$
  2. Choose an alternative uniformly at random from  $X$
- With probability 1/2:
  1. Choose an alternative uniformly at random from  $A$

## Proportional Fairness

**Definition 2 (Proportional Fairness).** The proportional fairness of a voting rule  $f$  is

$$PF_m(f) = \max_{\vec{\sigma} \text{ induced by } \vec{u}} \max_{\mathbf{y} \in \Delta(A)} \frac{1}{n} \sum_{i \in N} \frac{u_i(\mathbf{y})}{u_i(f(\vec{\sigma}))},$$

i.e., the maximum possible **average multiplicative increase in agent utilities** when moving from  $f(\vec{\sigma})$  to any other  $\mathbf{y}$ .

- First proposed in communication networks [Kelly et al., 1998].
- Scale-invariant: value stays the same if we rescale utility functions.

## Optimal Proportional Fairness

**Theorem 2.** There **exists** a voting rule which is  **$O(\log m)$ -proportionally fair**.

- Our existence proof uses the minimax theorem for zero-sum games.

**Theorem 3.** This bound is **optimal**, as there are preference profiles for which any distribution has no approximation better than  **$\Omega(\log m)$** .

**Theorem 4.** Given a preference profile, we can **compute in polynomial time** a distribution with an (almost) **optimal approximation** to proportional fairness.

## Proportional Fairness $\Rightarrow$ Nash Welfare Distortion

**Nash Welfare Distortion.** We also study the Nash welfare, which is the geometric mean of agent utilities:  $NW(\mathbf{x}, \vec{u}) = (\prod_{i \in N} u_i(\mathbf{x}))^{1/n}$ .

We can define **distortion with Nash welfare** by replacing UW in Definition 1 with NW.

- It is well-known that  $\text{Dist}_m^{\text{NW}}(f) \leq PF_m(f)$ .  
 $\Rightarrow$  there exists a voting rule  $f$  with

$$\text{Dist}_m^{\text{NW}}(f) \leq O(\log m).$$

- Our lower bound is at most  $e \approx 2.718$ .

**Open Problem:** Is constant distortion with Nash welfare achievable?

## Truthful Voting Rules

**Harmonic Rule** [Boutilier et al., 2015]:

- With probability 1/2:
  1. Select an *agent*  $i$  uniformly at random
  2. Choose agent  $i$ 's  $k^{\text{th}}$  most preferred alternative with probability  $\propto 1/k$
- With probability 1/2:
  1. Choose an alternative uniformly at random from  $A$

	Distortion, unit-sum	Distortion, unit-range	Proportional fairness
Harmonic Rule (truthful)	$\Theta(\sqrt{m \log m})^*$	$\Theta(m^{2/3} \log^{1/3} m)$	$\Theta(\sqrt{m \log m})$
Best possible, truthful	$\Theta(\sqrt{m \log m})^\dagger$	$\Theta(m^{2/3})^\ddagger$	$\Omega(\sqrt{m}), O(\sqrt{m \log m})$

\* [Boutilier et al., 2015],  $\dagger$  [Bhaskar et al., 2018],  $\ddagger$  [Filos-Ratsikas et al., 2014, Lee 2013]

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