On the Robustness of Second-Price Auctions in Prior-Independent Mechanism Design

Jerry Anunrojwong, Santiago Balseiro, Omar Besbes

Columbia Business School

Motivation

- mech design: how to optimally sell things
- classical theory too detail-dependent, so we **relax the common prior assumption** (Wilson doctrine)

Problem Formulation

- Optimize over direct mechanisms (x, p) selling one indivisible item to n buyers.
- mechanism is **prior-independent**
  - no need to know F ("detail-free" or "robust")
  - performance guarantee over all $F \in \mathcal{F}$
- We consider many dist classes on $[0,1]^n$.
- **dominant strategy** IC+IR
  - each buyer need not know other buyers' dists
- Objective = "regret" on revenue
- Benchmark = maximum possible revenue when valuation is known = $\max(\boldsymbol{v})$.

Research question:
What is an optimal detail-free mechanism and how well can we perform?

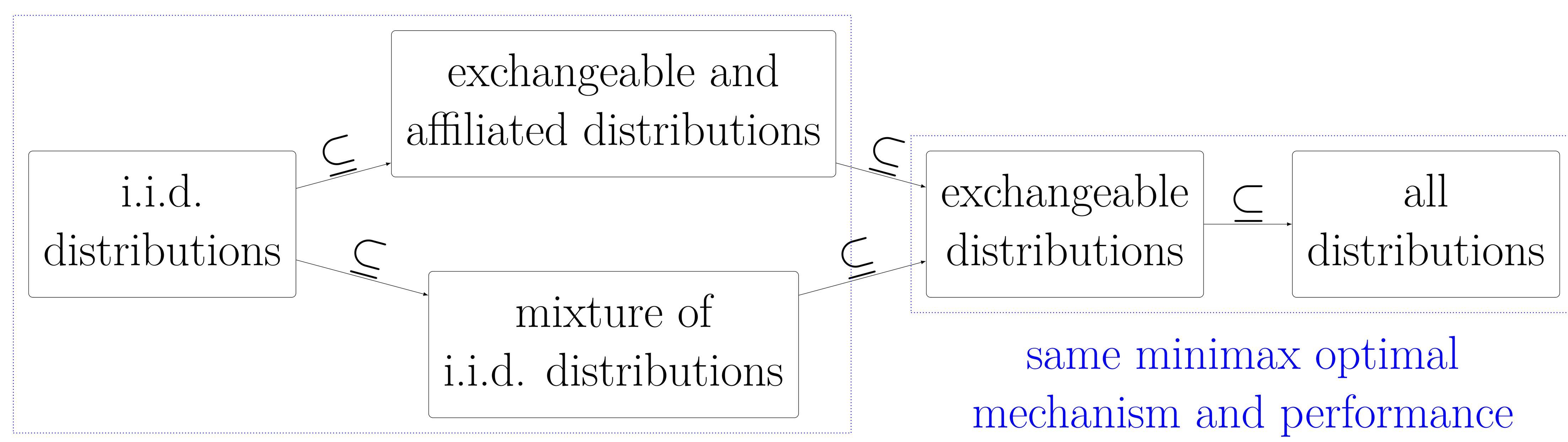
Challenges

- The space of all mechanisms is large.
- The space of all bounded dists is large.
- The problem is **nonconvex** due to class restriction in $\mathcal{F}$ e.g. i.i.d.

Minimax Problem For Each Distribution Class $\mathcal{F}$

$$\min_{\substack{\text{mech } (x,p) \\ \text{IC+IR}}} \max_{\boldsymbol{F} \in \mathcal{F}} \mathbb{E}_{\boldsymbol{v} \sim \boldsymbol{F}} \Bigg[\underbrace{\max(\boldsymbol{v})}_{\text{benchmark}} - \underbrace{\sum_{i=1}^{n} p_i(\boldsymbol{v})}_{\text{revenue}} \Bigg]$$

Main Result



Second Price Auction with Random Reserve is minimax optimal across many distribution classes!

Theorem

Under the distribution class of {i.i.d., mixture of i.i.d., exchangeable and affiliated}, the minimax regret admits as an optimal mechanism a second-price auction with random reserve price with cumulative distribution Φ_n^* on $[r_n^*, 1]$ given by

$$\Phi_n^*(v) = \left(\frac{v}{v - r_n^*}\right)^{n-1} \log\left(\frac{v}{r_n^*}\right) - \sum_{k=1}^{n-1} \frac{1}{k} \left(\frac{v}{v - r_n^*}\right)^{n-1-k},$$

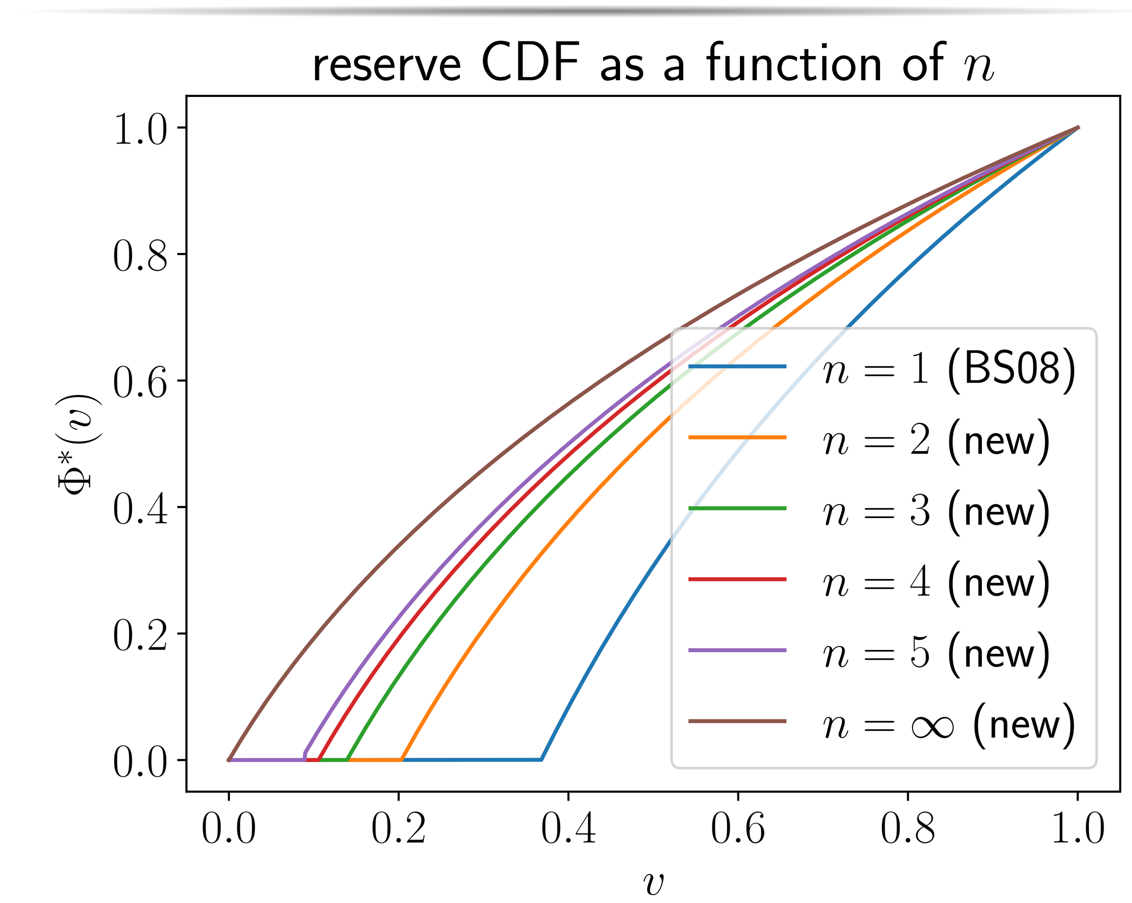
where $r_n^* \in (0, 1/n)$ is the unique solution to

$$(1 - r^*)^{n-1} + \log(r^*) + \sum_{k=1}^{n-1} \frac{(1 - r^*)^k}{k} = 0.$$

Our Approach

- **saddle point approach**: find m^*, F^* $R(m^*, \boldsymbol{F}) \le R(m^*, \boldsymbol{F}^*) \le R(m, \boldsymbol{F}^*)\ \forall m, \boldsymbol{F}$
- Conjecture that $m^* = \text{SPA}(\Phi^*)$.
- Φ^* minimizes $R(\Phi, F^*)$, which is linear in $\Phi \Rightarrow$ pins down F^*
- F^* maximizes $R(\Phi^*, F)$ which is a function of $F(\cdot) \Rightarrow$ pins down Φ^*

Insights



n	OPT	SPA(0)	SPA(r^*)
1	0.3679	1.0000	0.5000
2	0.3238	0.5000	0.4444
3	0.3093	0.4444	0.4219
4	0.3021	0.4219	0.4096
5	0.2979	0.4096	0.4019
10	0.2896	0.3874	0.3855
∞	0.2815	0.3679	0.3679

- value of competition positive as $n \to \infty$
- significantly outperforms benchmarks (no & optimal deterministic reserves)