Linear Pricing Mechanisms for Markets without Convexity

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Setting

Two-sided, quasilinear markets with L commodities
N buyers, $F = \phi N$ sellers, with fixed costs or other non-convexities

Walrasian equilibria may not exist!

Challenges

1. Computational complexity
2. Balancing budgets
3. Achieving individual rationality
4. Ensuring truthful reporting despite rationing

Bound-form First Welfare Theorem

Given **any** feasible allocation $\omega = (x, y)$, **any** nonnegative prices $p \in \mathbb{R}^L_+$, the surplus function $S: \Omega \to \mathbb{R}$, and an efficient $\omega^* \in \Omega$,

$$S(\omega^*) - S(\omega) \leq p \cdot \left(\sum_f y_f - \sum_n x_n \right) + \left(\sum_f \mathcal{R}_f(p, y_f) + \sum_n \mathcal{R}_n(p, x_n) \right)$$

loss at ω $\leq$ budget deficit $+$ sum of **rationing losses**

An agent's **rationing loss** at price p and allocation ω is the difference between their *maximal* payoff at p and their *realized* payoff at (p, ω).

Two new extensions of the Walrasian mechanism

Markup Mechanism (α, p, ω)
Sellers paid prices p and buyers pay prices $(1+\alpha)p$, allocation ω in supply and demand sets at respective prices, while enforcing physical and budget feasibility

for markets with two-sided non-convexities

Rationing Mechanism (p, ω)
A single price p is used for payments, but some agents are **rationed** to ensure ω has no unallocated supply

for markets with one convex side

If Walrasian equilibria exist, both coincide with Walrasian mechanism.

We focus on two simple-to-compute instances of these mechanisms requiring only *convex* optimizations and a single binary search.

Efficiency

Markup mechanism: can choose $\alpha \sim O(1/N)$ leading to $O(1/N)$ percentage welfare loss
Rationing mechanism: can identify p with $O(1/N)$ total loss under strong monotonicity assumption

Incentives

Both mechanisms have similar incentive properties to Walrasian mech, but rationing mech may be $\varepsilon - IR$

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