Is Selling Complete Information (Approximately) Optimal?

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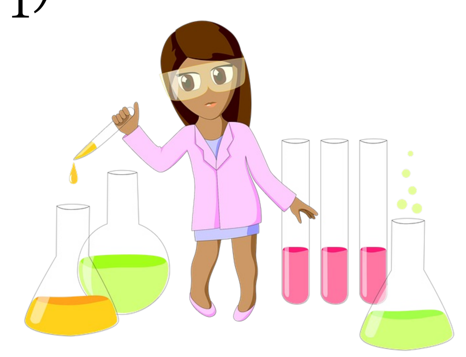
Model

- ❑ $\Omega = [n]$: set of states, $A = [m]$: set of actions
- ❑ A single data buyer has payoff $u_{\omega,a}$ for every state ω and action a.
- ❑ Buyer has prior belief θ about the state of the world → type
- ❑ Type θ is drawn from distribution D.
- ❑ Seller reveals information through signaling schemes - Experiments

$$\Omega = \{sick, healthy\}, A = \{\text{home}, party\}, \text{u} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



θ: prior prob of being sick

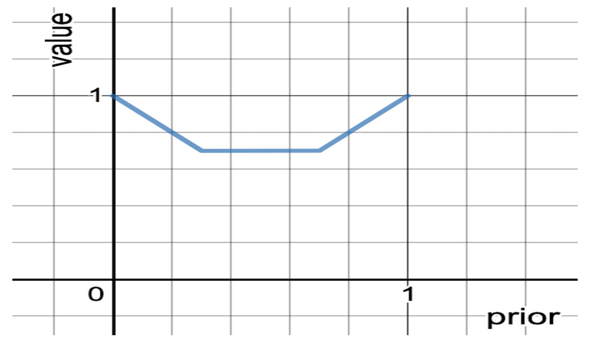
	positive	negative	
Test: E =	0.7	0.3	sick
	0.3	0.7	healthy

Procedure for any mechanism:

1. Seller commits to a mechanism M.
2. The state of the world ω and buyer type θ are realized.
3. Buyer reports her type to the mechanism.
4. Seller sends the signal according to experiment $E(\theta)$ and state ω.
5. Buyer chooses an action a, obtains payoff $u_{\omega,a}$, and pays a payment $\text{t}(\theta)$ to the mechanism.

- ❑ After receiving the signal: The buyer performs a Bayesian update on her belief, then chooses the best action that maximizes her expected payoff according to the posterior.
- ❑ Buyer's value for an experiment: A piecewise linear function

$$V_\theta(E) = \sum_k \max_a \sum_\omega \theta_\omega E_{\omega,k}\, u_{\omega,a}$$



- ❑ The revenue-optimal mechanism is achieved by a menu of experiments.

Test A: \$0.1 $\begin{pmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{pmatrix}$ Test B: \$0.2 $\begin{pmatrix} 0.8 & 0.2 \\ 0.25 & 0.75 \end{pmatrix}$ Test C: \$0.25 $\begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix}$

- ❑ [Bergemann, Bonatti, Smolin '18]: Characterize the optimal menu when there are 2 states, 2 actions. The optimal menu contains at most 2 experiments.

Single-Dimensional Setting!

Our paper:

- ➢ Consider more general settings (2 states + m actions / n states).
- ➢ Study the simplest menu: Selling complete information
- ➢ "Simple vs. Optimal" – How large is the revenue of selling complete information compared to the optimal menu?

"Perfect Test" $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Main Result (2 States)

Result 1: In 2 states + m actions, $\frac{\text{OPT}}{\text{FRev}} = O(m)$. The ratio is tight up to a constant.

FRev – best selling complete information mechanism
OPT – optimal menu

Important corollary: The optimal menu has cardinality $\Omega(m)$.

- ❑ Useful Lemma: Given any menu M with cardinality k, we can achieve revenue $\geq \frac{1}{k} \cdot Rev(M)$ by selling complete information.

Challenges and Proof Idea

Selling Information vs. Auctions:

- ❑ Different types interpret the signal differently → Buyer's value function for any fixed experiment is not linear in the type, but piecewise-linear.
- ❑ IC constraints are more demanding.
  - ❑ Standard IC: After deviating to another type, the buyer has to follow the seller's recommendation (wlog assume the signal set is the action set).
  - ❑ σ-IC: Deviation from both the true type and the seller's recommendation. "Double Deviation"
- ❑ IR constraints are more demanding.
  - ❑ Initially, buyer has positive utility before receiving any information ($\max\{\theta, 1-\theta\}$ in the example).

Main Challenge: When there are 2 actions, σ-IC is implied by standard IC + IR. Not the case for $m \geq 3$ actions.

Proof sketch for upper bound:

1. Relax the problem by dropping the σ-IC constraints.
2. Show that the optimal menu of the relaxed problem has cardinality $O(m)$.
   - ➢ The "relaxed" optimal is achieved by a special class – "semi-informative menu"

(a) Pattern 1: $\pi_{2m}(E) = 1$

E	1	$\cdots$	j	$\cdots$	m
ω_1	π_{11}	$\cdots$	0	$\cdots$	$1-\pi_{11}$
ω_2	0	$\cdots$	0	$\cdots$	1

(b) Pattern 2: $\pi_{11}(E) = 1$

E	1	$\cdots$	j	$\cdots$	m
ω_1	1	$\cdots$	0	$\cdots$	0
ω_2	$1-\pi_{2m}$	$\cdots$	0	$\cdots$	π_{2m}

Table 2. Two Specific Patterns of the Semi-informative Experiment

3. By the useful lemma (above), $\frac{\text{OPT}}{\text{FRev}} = O(m)$.

For the relaxed problem:

- ➢ We show a characterization of the optimal menu for the relaxed problem.
- ➢ Corollary: A sufficient condition under which selling complete information is optimal.

$n \geq 3$ States

Focus on a special environment: 3 states, 3 actions, $u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ "matching utility"

Result 2:

- ➢ Even in this special environment, there exists a distribution D such that $\frac{\text{OPT}}{\text{FRev}} = \Omega(poly(N))$. $N = |supp(D)|$
- ➢ Corollary: There is no universal finite upper bound for the cardinality of the optimal menu that works for all distribution D.

- ❑ Proof is adapted from [Hart and Nisan '19] for auctions.

Intuition: $E = \begin{pmatrix} x_1 & 0 & 1-x_1 \\ 0 & x_2 & 1-x_2 \\ 0 & 0 & 1 \end{pmatrix}$

Buyer's value for E if she follows the recommendation: $\underbrace{\boxed{\theta_1 x_1 + \theta_2 x_2}}_{\text{value function for 2 items}} + (1-\theta_1-\theta_2)$

- ❑ Design the discrete types carefully such that
  - ❑ Buyer at each type will follow the recommendation after choosing the experiment.
  - ❑ The more demanding IC, IR constraints are satisfied.
- ❑ Follow-up: What if D is a special distribution?

Result 3: If D is a uniform distribution, then selling complete information is optimal.

Open Questions

- ❑ For binary states, can we show an upper bound (in # actions) on the cardinality of the optimal menu?
- ❑ Other simple mechanisms (eg. semi-informative menu) that obtain a better approximation?
- ❑ ArXiv version of the paper: https://arxiv.org/abs/2202.09013

Thank you for your time!