Core-Stability in Assignment Markets with Financially Constrained Buyers

Eleni Batziou Martin Bichler Maximilian Fichtl

Dept. of Computer Science, Technical University of Munich (TUM), Germany

Motivation

- In real economic transactions, participants do not have access to unlimited funds
- Without financial constraints, nice properties such as envy-freeness, core-stability and welfare-maximization can be achieved by truthful auctions in polynomial time
- Existence of budgets complicates efficient assignment computation, and we must therefore prioritize
- In combinatorial auctions, finding core-stable assignments when buyers are payoff-maximizing is Σ_2^p-hard: Bichler and Waldherr (2020)

Contribution

- We study properties achievable in assignment markets with unit-demand buyers that are payoff-maximizing but have hard budget constraints
- We design an ascending auction that always results in a core-stable outcome, using only demand queries
- With appropriate decisions made throughout the process, the resulting outcome is additionally welfare-maximizing
- We prove that no auction mechanism can satisfy incentive-compatibility and core-stability in assignment markets with budgets
- Under full information, we design a Mixed Integer Program (MIP) that retrieves the desired assignment
- We prove that the computational problem is NP-hard given budget-constrained buyers

Model

- A two-sided matching market is described by the quintuple $G = (\mathcal{B}, \mathcal{S}, v, b, r)$.
- Set of buyers $i \in \mathcal{B} = \{1, \ldots, n\}$
- Set of goods $j \in \mathcal{S} = \{1, \ldots, m\} \cup \{0\}$ - each good identified with seller that owns it
- Buyer i's valuation $v_i = S \to \mathbb{Z}_{\geq 0}$
- Buyer i's budget $b_i \in \mathbb{Z}_{\geq 0}$
- Seller j's reserve value $r_j \in \mathbb{Z}_{\geq 0}$
- Price vector $p \in \mathbb{R}^{\mathcal{S}}$, with $p(j)$ assigned to good $j \in \mathcal{S}$
- Quasi-linear utility up to budget: $\pi_i(j,p) = v_i(j) - p(j)$ if $p(j) \leq b_i$, or $\pi(j,p) = -\infty$ o/w
- Assignment $\mu : \mathcal{B} \to \mathcal{S}$ - Outcome consists of tuple (μ, p)

Core-Stability

- Blocking pair: buyer-seller pair (i,j) for which it holds that $\pi_i(j,p) > \pi_i(\mu(i),p)$ and $p(j) < b_i$, given outcome (μ, p)
- An outcome is defined as **core-stable** if there does not exist any possible blocking pair of agents
- There is no feasible coalition of agents all of which have higher payoff under any alternative assignment
- The notion of the core coincides with that of a competitive equilibrium, when budgets are not present in the auction

Iterative Auction

- We base our algorithm on the well-known auction by Demange et al. (1986), which implements the Hungarian algorithm
- The algorithm does not have access the the full valuation profiles, only queries the demand set of each buyer
- We define the restricted demand set of each bidder R_i - set of affordable items that maximize utility
- Additionally, we use the usual notions of (minimally) over- and underdemanded sets
- The goal of our auction is to determine prices and sets R_i, such that there are neither over- or underdemanded sets of items

Auction Algorithm

1. Round $t := 1$: Set all prices to 0 and all sets to empty. Restricted demand set of each i is $\mathcal{S}$.
2. Request all demand sets. Define I^t as the set of all bidders that have reached their budget constraint for some item at price p^t. If $I^t \neq \emptyset$, go to Step 3. Otherwise, if overdemanded set exists, go to Step 4. Else, go to Step 5.
3. Choose bidder i from I^t, remove item that caused to reach budget from R_i, and set prices $p^{t+1} := p^{t-1}$. Set $t := t+1$ and go to Step 2.
4. Choose minimally overdemanded set O^t, raise prices for items in set, increment round $t := t+1$ and go to Step 2.
5. Compute μ such that all assigned bidders have demand for their goods and can afford them. Return tuple (μ, p) and terminate the auction.

Economic Properties

- The auction terminates after a finite number of iterations, and the output (μ, p) of Step 5 constitutes a core-outcome
- If every time Step 3 is executed, $|I|^t = 1$, and there is a unique outcome after Step 5, then it is *bidder-optimal* and thus welfare-maximizing

Incentive Compatibility

- General position introduced by Aggarwal et al. (2009): In an ascending auction, no two bidders can reach their budget limits at the same time
- If the auction is in general position, it is ex-post incentive compatible
- For the general case of assignment markets with payoff-maximizing but budget constrained buyers, we prove that there is no incentive compatible mechanism terminating in a core-stable solution for every input

Computational Complexity

- Binding budgets: valuations for items are larger than budget constraints
- W.l.o.g. we assume that winning buyers pay price equal to their budget
- We prove that the problem is NP-hard via a poly-time reduction from the Maximum Independent Set problem for cubic graphs
- The proof uses vertex and edge gadgets, where buyer and seller sets represent the two sides of the bipartite graph
- Edge weights define valuations and prices
- In edge gadgets, there are two possible stable matches of equal welfare, while in vertex gadgets, their welfare differs by 1
- Maximum welfare on the graph has an added factor k, which corresponds to the size of the independent set
- Our objective is to determine a stable matching in the transformed graph that maximizes welfare

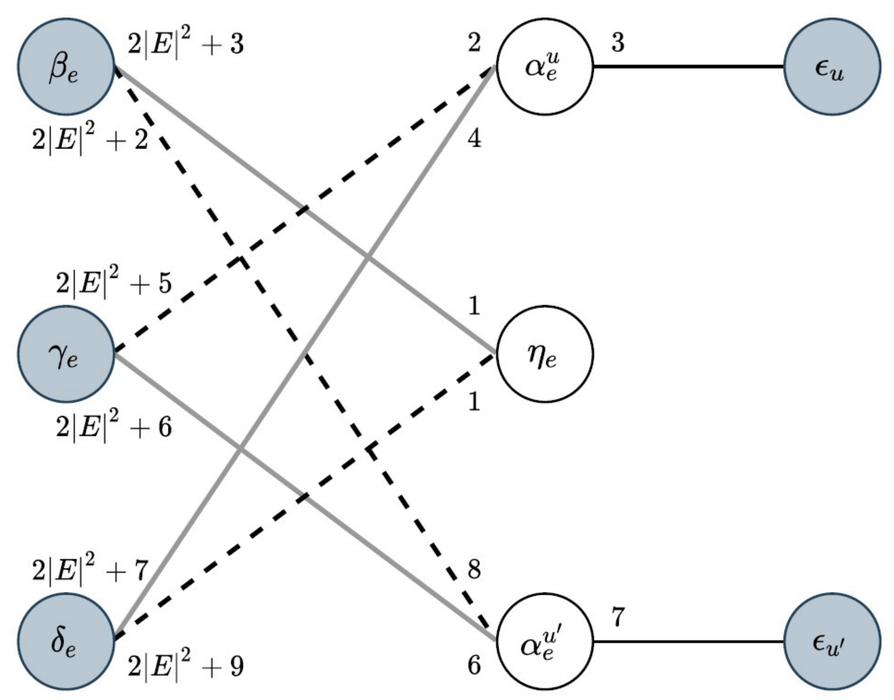

Figure: Edge Gadget.

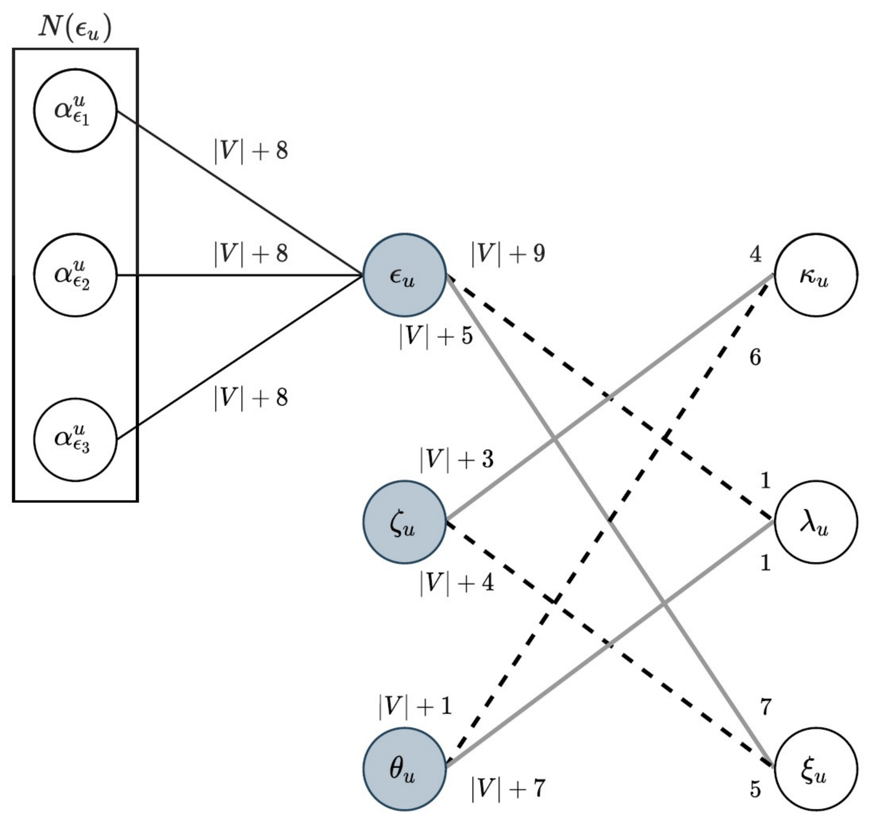

Figure: Vertex Gadget.