The Value of Excess Supply in Spatial Matching Markets

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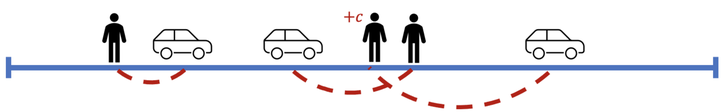
Model

- Time is discrete
- n drivers distributed uniformly at random on $[0, 1]$
- m riders arrive one in each period, also distributed uniformly *(possibly adversarial order)*.
- The platform assigns rider to a driver irrevocably
- Or it can wait with cost $+c$ per period, until it assigns a driver
- Cost of matching rider at location r to driver at d is $|r - d|$.



Creating Unblancedness vs. Improving Matching

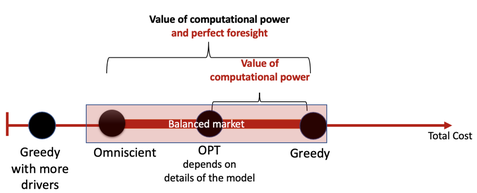
the value of computational sophistication and foresight in a *balanced* market

vs.

the cost of increasing the supply of drivers; Creating unbalanced market

Two Benchmarks

- **Omniscient** matching algorithm has perfect foresight of the future arrivals and chooses the ex post best matching.
  - It is better than OPT.
  - It matches riders immediately (no waiting cost)
  - "Detail-free"
- **Greedy** matching algorithm matches each rider upon arrival to the closest driver.
  - It's naive!
  - It matches riders immediately (no waiting cost)
  - "Detail-free"



Perfect Forecast in a Balanced Market

Balanced market consists of n riders and n drivers.

Observation (Balanced Market)

Let $R = \{r_1, \ldots, r_n\}$ and $D = \{d_1, \ldots, d_n\}$ be two sets random points drawn independently from a uniform distribution on $[0, \ell]$. Then there exists $C_1, C_2 > 0$

$$C_2\sqrt{n} \geq \mathbb{E}[cost(OMN)] \geq C_1\sqrt{n}.$$

Greedy in an Unbalanced Market

Unbalanced market consists of n riders and m drivers.

Theorem (Greedy)

Let $R = \{r_1, \ldots, r_m\}$ and $D = \{d_1, \ldots, d_n\}$ be two sets random points drawn independently from a uniform distribution on $[0, \ell]$. Then there exists $C > 0$,

$$\mathbb{E}[cost(Greedy)] \leq C \log^3(n) \frac{(m+n)^4}{(m-n)^4}.$$

Corollary (Unbalanced Market)

Consider an unbalanced market where $m \geq (1+\epsilon)n$, for some $\epsilon > 0$. Then, there exists some $C_\epsilon > 0$ such that for large enough n

$$\mathbb{E}[cost(Greedy)] \leq C_\epsilon \log^3(n).$$

The number of extra drivers in simulation

Simulations show that for $n = 100$, with 4 more drivers greedy beats omniscient in the balanced market. Also, for $n = 1000$ only 13 more drivers are enough

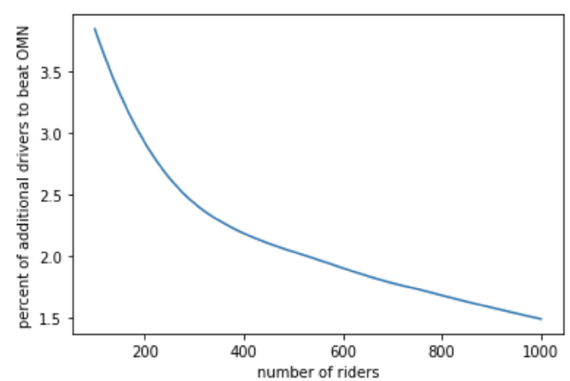
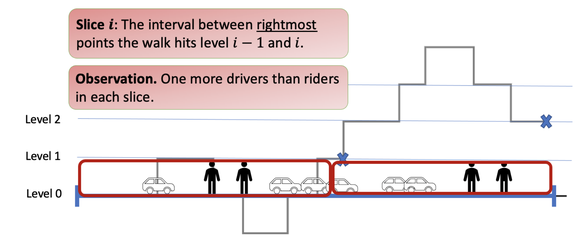


Figure 1. Additional percentage of drivers required for greedy to outperform omniscient.

Proof Overview: Random Walk Representation

Partition the interval into small slices such that:

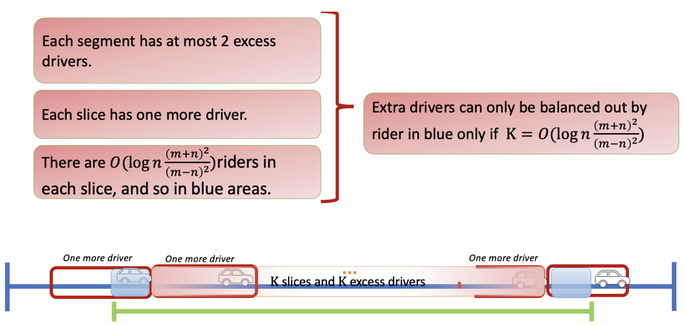
1. whp $O(\log n \frac{(m+n)^2}{(m-n)^2})$ riders in each slice every slice contains one more driver *(this condition upper bounds the length of each slice)*.
2. every slice contains one more driver



Proof Overview (2): Greedy doesn't Skip Many Slices

Run the procedure along with greedy:

1. start with segments between consecutive drivers
2. when a rider arrives, merge segments that overlap with matching



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